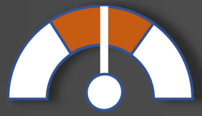
Confidence Intervals

Become Future Fit

You will learn

Learn what are Confidence Intervals & why should you know about it

Level of Difficulty



Medium

Confidence Intervals

It represents the uncertainty in the estimation of true mean(or even the variation).

Confidence Intervals

Sample statistic ± [___ X ___]

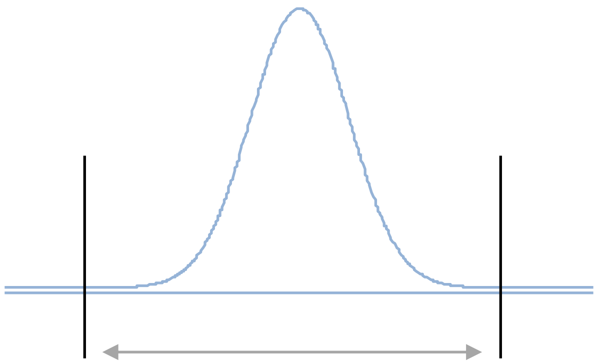
- e.g., $\bar{x}$, s, …
- Confidence factor
- Measure of variability

In some cases the uncertainty is not symmetrical and the ‘+’ term is different from the ‘-’ term; e.g., for σ.

Confidence Intervals

Confidence Intervals are nothing but upper and lower limits of a band within which we find the true mean.

Area under the Normal Curve



99% = 3σ

95.45 % = 2σ

% Area under Curve = $Z\sigma$

Central Limit Theorem

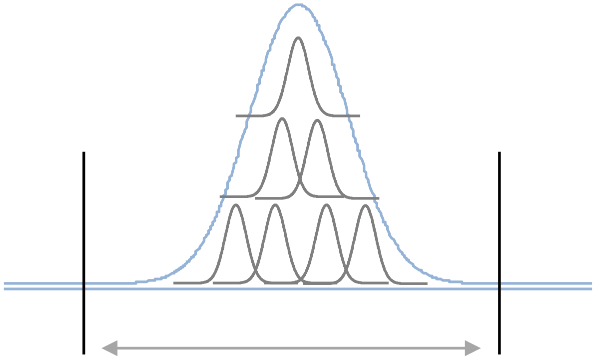
The standard deviation of the SAMPLE MEAN:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard Error of Mean (SE)

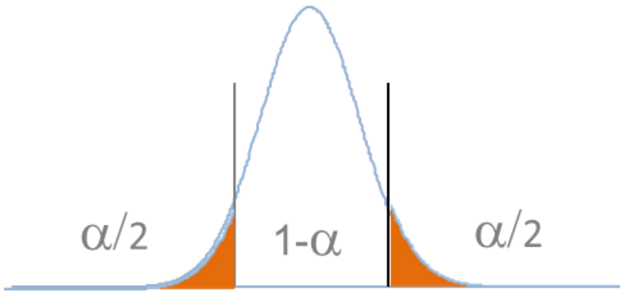
Confidence Intervals

Population Individual Values Vs Sample Means Values



% Area under Curve = $Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

Confidence Intervals



Population Individual Values Vs Sample Means Values

% Area under Cuve = $Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

Confidence Interval for Means

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Learn how to compute Confidence Intervals for means

Level of Difficulty

Medium

CI for Mean of Continuous Data

$$\mathbf{CI} = \overline{\mathbf{X}} \pm \mathbf{Z}_{\alpha/2}\left(\frac{\boldsymbol{\sigma}}{\sqrt{\mathbf{n}}}\right)$$

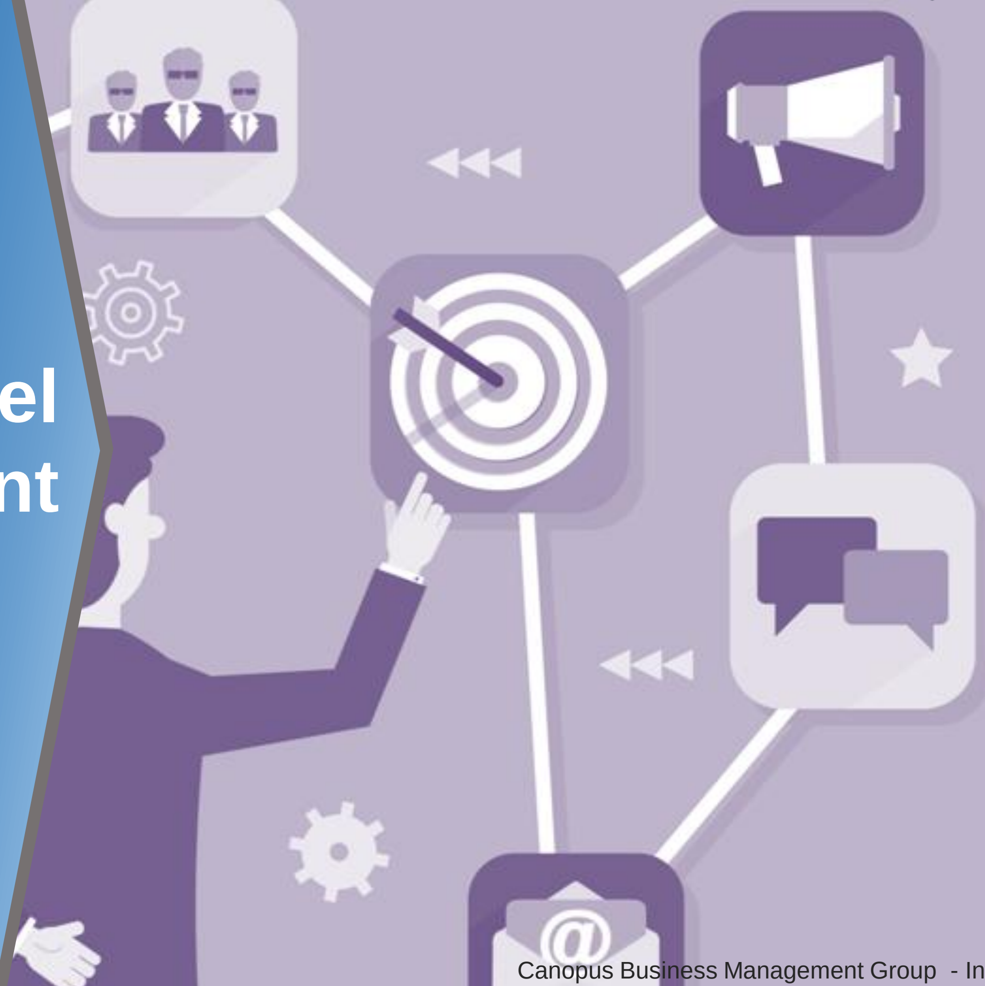
$Z_{\alpha/2}$ = normal distribution value for a given confidence level

$\overline{X}$ = mean of data

σ = population standard deviation

n = sample size

Competency Level Assessment

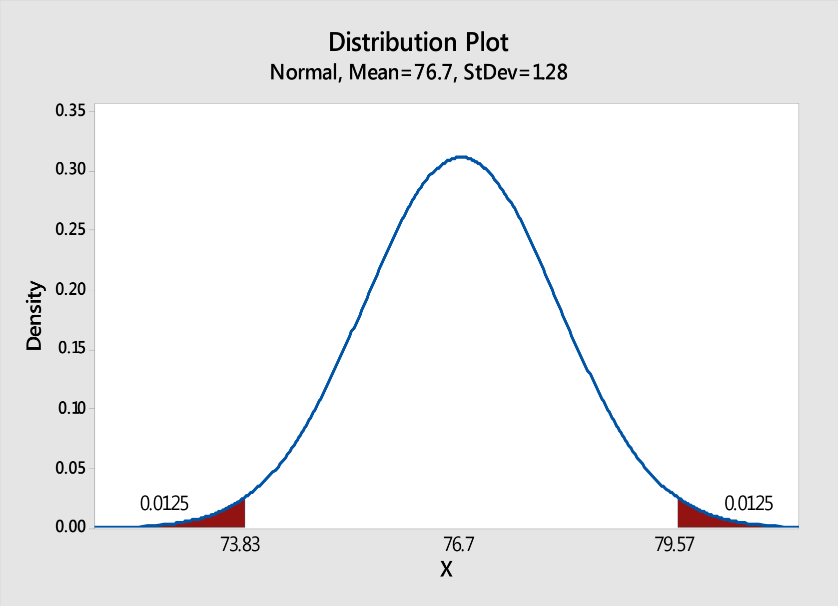


Competency Level Assessment

Company has established norms for the competency of their executives in aptitude test related to a specific subject. The historic population data suggests that average 73.2 with a standard deviation of 8.6.

If 45 randomly selected persons and have a average 76.7, find the confidence interval of the scores at 95% level of confidence

CI for Mean of Continuous Data



CI for Mean of Continuous Data

- This formula only applies when σ is known, which is rare.
- If the sample size is large (exceeds 50), it is a good approximation.

Degrees of Freedom

Become Future Fit

You will learn

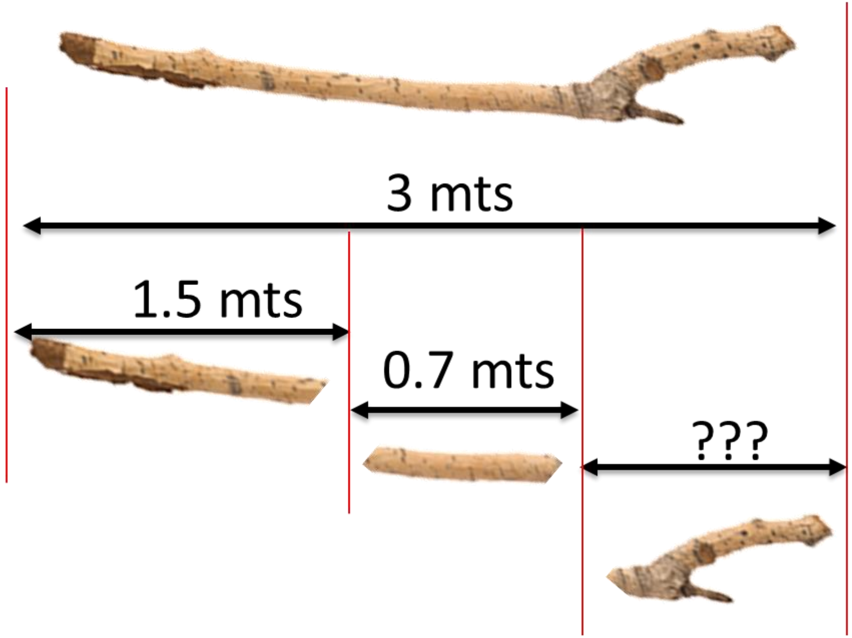
Learn about what is Degrees of Freedom &

Where will it be applied

Level of Difficulty

Medium

Degrees of Freedom



Degrees of Freedom

The number of independent pieces of information that go into the estimate of a parameter is called the degrees of freedom

Degrees of Freedom

DoF is the number of chances we have to determine the characteristics of a system.

Degrees of Freedom

Example

- If there are 50 samples, then the degree of freedom will be 49
- If there are 5 groups/levels, the degree of freedom will be 4

Sampling Distribution

1. t Distribution
2. F Distribution
3. Chi-Square Distribution

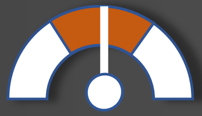
t Distribution

Become Future Fit

You will learn

Learn about t distribution & its relevance to data analysis

Level of Difficulty



Medium

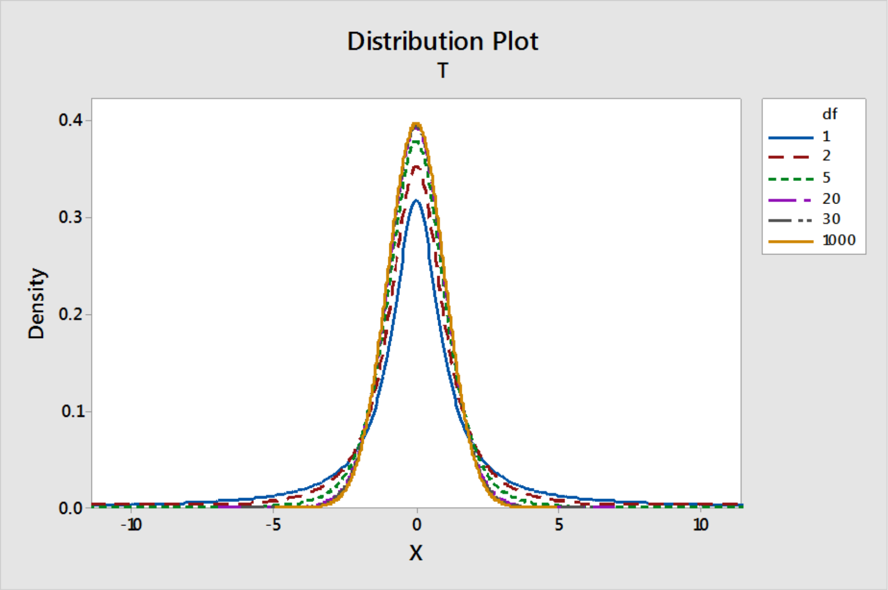
When the Sample Size < 30

&

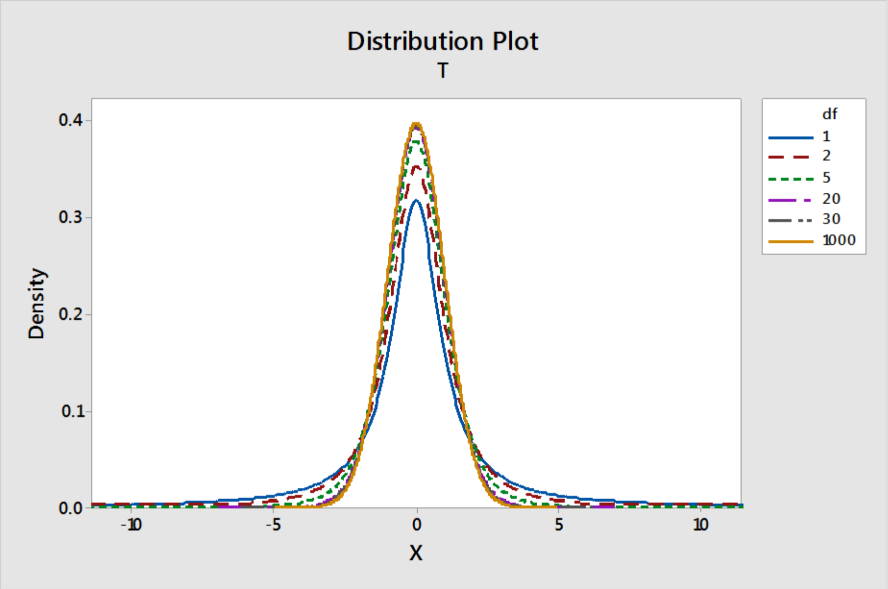
Population SD is not known

Students t-Distribution

t-Distribution

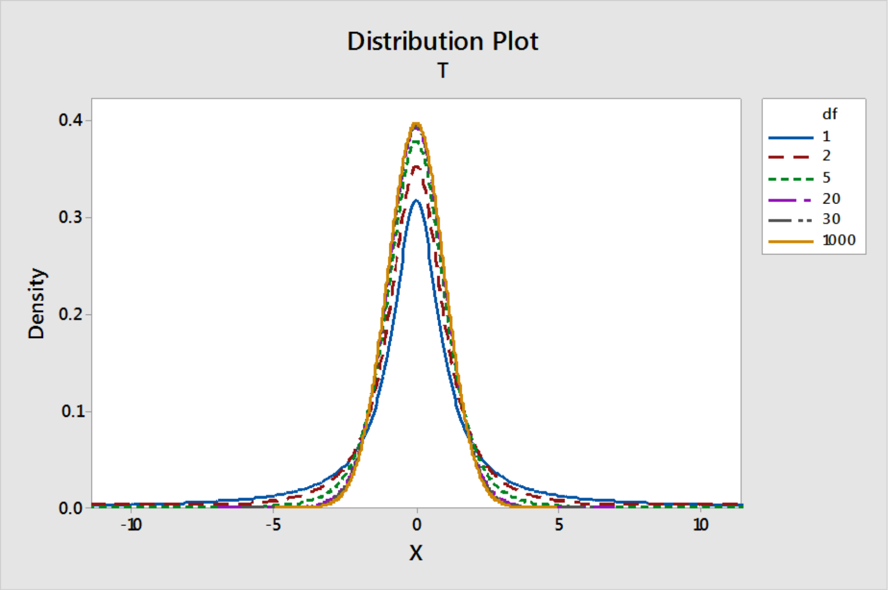


t-Distribution



- Looks like Normal Distribution
- Has fatter tails

t-Distribution



Developed in 1908 by William Gosset

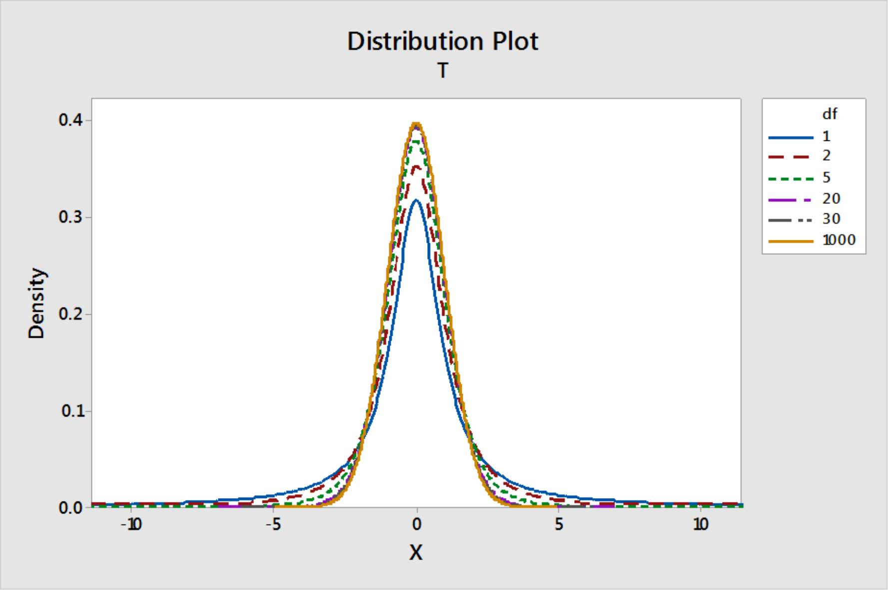
Properties

- Mean of t-distribution = 0
- Variance = $\nu / (\nu - 2)$,
- Where ν is the degrees of freedom
- t-distribution Shape & Scale parameters are dependent on Degrees of Freedom

Properties

- Always Variance > 1
- When ν ~ Large
  - Variation ~ 1 $[\nu / (\nu - 2)]$
  - t Distribution → Standard Normal Distribution

Fatter Tails



t- statistic

Sample Size	t-value ($\alpha/2$ =.025)
2	12.71
3	4.30
5	2.78
10	2.26
20	2.09
30	2.05
100	1.98
1000	1.96

When to use t-Distribution

Use t distribution when population distribution is normal or any bell-shaped distribution & sampling distribution has below characteristics

Sample	Characteristics
< 15	Sampling distribution is symmetric, unimodal, no outliers
16~40	Sampling distribution is moderately skewed, unimodal, without outliers
> 40	Sampling distribution doesn’t have outliers.

Don’t use t distribution when population is not approximately normal

Confidence Intervals for t Distribution

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You will learn

Learn to compute the Confidence Limits for sample mean using t-distribution

Level of Difficulty

Medium

Confidence Intervals for Sample Data

Inspection of 10 items gave the following measurements :

0.983, 1.002, 0.998, 0.996, 1.002, 0.983, 0.994, 0.991, 1.005 & 0.986.

What is the likely variation in the measurements with 99% confidence ?

CI with t Distribution

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Formula

$$CI = \overline{X} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}$$

$\overline{X}$ = Mean

s = Standard deviation

n = Sample size

υ = Degrees of freedom, used in some tables & calculated as n-1 for this test.

$t_{\alpha/2,n-1}$ = Value from t distribution

Confidence Interval for Proportions

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Learn to compute Confidence Interval for proportion data

Level of Difficulty

Medium

Scenario

An internal auditor randomly selects 200 documents for an audit. He finds 5 defective documents. Calculate a 90% confidence interval for the proportion of good documents.

Assumptions

If n> 5/min(p,q)

Binomial tends to Normal

Mean (μ) = n*p ; SD (σ) = sqrt[n*p*q]

CI for Proportions

Discrete Data

$$\bar{p} \pm Z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Where…

$\bar{p}$ = average proportion seen in the sample

n = sample size

α = risk

CI for Proportions

Discrete Data

$$\bar{p} \pm Z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Binomial Approximates to Normal Distribution for large Samples.

Use for large sample sizes

Chi-square Distribution

Become Future Fit

You will learn

Learn about Chi-square distribution & its relevance to data analysis

Level of Difficulty

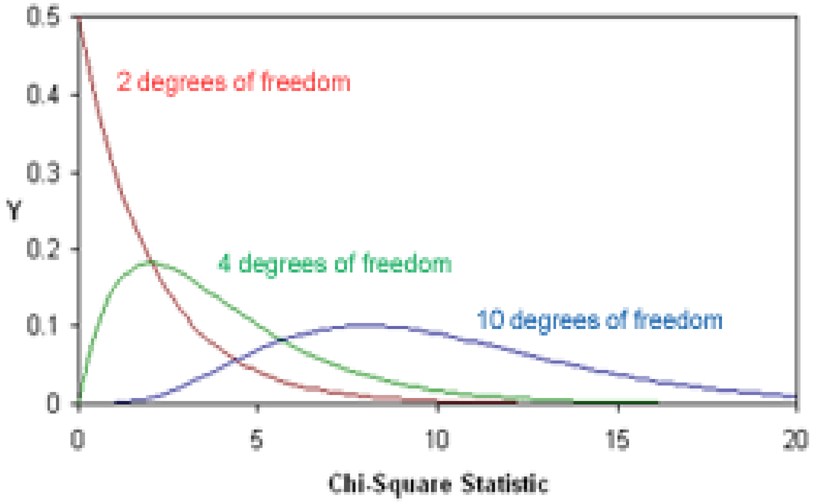
Medium

Chi-Squared Distribution

If a process has cycle time data that is normal, then the SUM OF SQUARES of its standard normal variable ($\mu = 0$ and $\sigma = 1$) forms a Chi-square distribution

Chi-Squared Distribution

Chi-square is a family of distributions



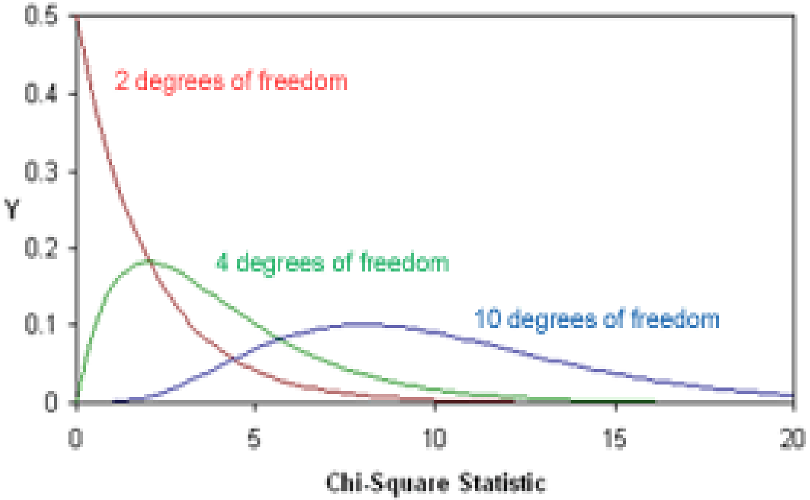
Chi-Squared Application

- For characterizing & analyzing dispersion of any data (Sum of Squares)
- For both Continuous & Discrete data:
  - Variance (Goodness of Fit)
  - Observed Vs Expected (Contingency Tables)

Properties

- Chi-square distribution has one parameter, degrees of freedom(ν)
  - Mean = ν
  - Standard Deviation = Sqrt(2ν)
- Chi-square is always a positive value

Properties



- As Degree of Freedom increases, Chi-square distribution tends to normal curve

CI for Variance

$$\frac{(n-1)s^2}{\chi^2_{(\alpha/2),(n-1)}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2),(n-1)}}$$

Where…

n = sample size

s^2 = variance

α = risk

CI will not be symmetrical on both sides

The standard deviation of process parameter is 0.86 mins as ascertained from 12 samples. Estimate 95% CI for Standard Deviation of this data.

Descriptive Statistics

N	StDev	Variance	95% CI for σ using Chi-Square
12	0.860	0.740	(0.609, 1.460)

F Distribution

Become Future Fit

You will learn

Learn about F Distribution and its relevance to data analysis

Level of Difficulty

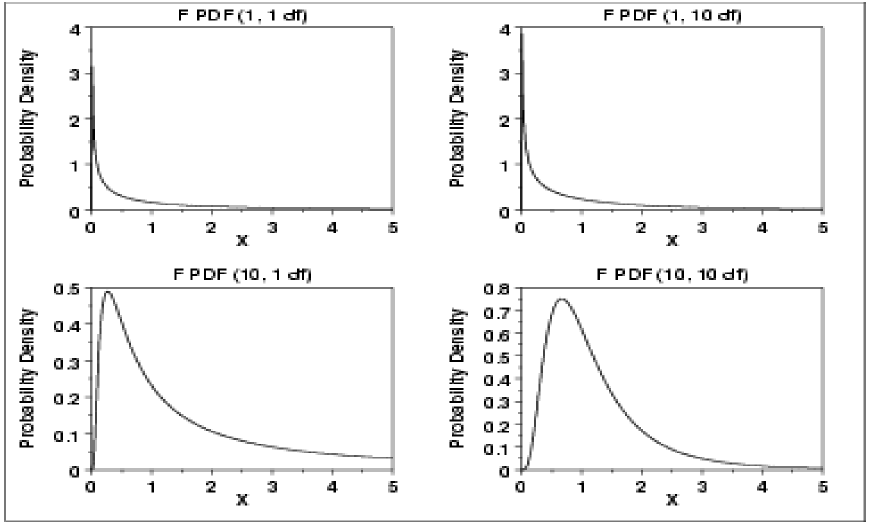
Medium

F-Distribution

The F distribution is the ratio of two chi-square distributions with degrees of freedom (n1-1) and (n2-1) respectively

F-Distribution

F Distribution is a family of distributions



F-Distribution

The ratio of two $\chi 2$ distribution with different degrees of freedom form a F distribution as below:

$$F_{\nu_1,\nu_2} = \frac{\chi_1^2/\nu_1}{\chi_2^2/\nu_2} \quad (\nu_1 \le \nu_2)$$

F-Distribution

Ratio of two sample variances from the same normally distributed population

$$F_{\nu_1,\nu_2} = \frac{s_1^2}{s_2^2} \quad (\nu_1 \le \nu_2)$$

Application

- Used to compare variances of different samples
- Many statistical tests are based on F test